On the Inevitability of Attractive Gravity in Variational Spacetime Theories

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Abstract

We examine whether the attractive sign of gravity is a contingent feature of nature or a structural necessity. Assuming (i) positive energy density and (ii) variational motion along spacetime geodesics, we show that repulsive gravity leads to geodesic defocusing, absence of bound solutions, and loss of physical structure at all scales. We argue that attractive gravity is not a choice of dynamics but a consistency condition required for the existence of stable trajectories, orbits, and temporal evolution. This result is independent of particle content and applies to any metric theory admitting a variational principle.

1 Introduction

While the mathematical structure of gravitational theories is well established, the physical necessity of gravity's attractive sign is rarely discussed explicitly. It is often treated as an empirical input rather than a structural requirement. In this note, we clarify why this sign cannot be reversed without eliminating bound structure altogether.

The attractive nature of gravity is typically motivated by observation: masses fall together, orbits close, and gravitational lensing converges. However, these are consequences rather than explanations. A more fundamental question is whether the sign of gravitational interaction is a free parameter that nature happens to select, or whether it is constrained by the internal consistency of physical theory itself.

We show that under minimal assumptions—positive energy density and variational geodesic motion—the repulsive case fails to admit bound solutions, stable structures, or persistent physical systems. This is not a statement about specific solutions or particular matter configurations, but about the geometric character of the theory as a whole.

2 Framework and Assumptions

To establish our argument on minimal grounds, we state our assumptions explicitly:

Assumption A (Variational Motion): Physical worldlines extremize proper time,

$$\delta \int ds = 0. \tag{1}$$

This is the geodesic principle, standard in all metric theories of gravity.

Assumption B (Positive Energy): The stress-energy tensor satisfies the weak energy condition,

$$T_{\mu\nu} u^\mu u^\nu \geq 0 \tag{2}$$

for all timelike vectors u^μ. This ensures that energy density is non-negative in all reference frames.

Assumption C (Metric Theory): Gravity is described by a spacetime metric $g_{\mu\nu}$ coupled to matter through Einstein's equations or a suitable generalization thereof.

No assumptions are made regarding extra dimensions, dark matter, or modified forces. The argument that follows is therefore independent of specific matter content and applies broadly to variational spacetime theories.

3 Newtonian Limit: Immediate Instability

Consider the Newtonian limit of gravity, governed by Poisson's equation:

$$\nabla^2\Phi = 4\pi G\rho. \tag{3}$$

In the standard attractive case, $G > 0$, and regions of positive mass density $\rho > 0$ produce a negative potential $\Phi < 0$. This admits stable bound orbits and equilibrium configurations.

Reversing the sign of G yields:

$$\nabla^2\Phi = -4\pi|G|\rho, \tag{4}$$

with $\Phi > 0$ for positive mass density. The resulting force law is repulsive, and the potential has no local minima. Consequently:

- No stable circular orbits exist.
- All initially bound systems become unbound.
- There are no equilibrium configurations for extended bodies.

The repulsive sign admits no bounded classical solutions, independent of initial conditions. While this argument is elementary, it is also fatal: a theory without bound states cannot sustain persistent physical structures.

4 Geometric Core: The Raychaudhuri Equation

The Newtonian analysis generalizes to full relativistic gravity through the Raychaudhuri equation, which governs the evolution of geodesic congruences. For a congruence of timelike geodesics with expansion scalar θ, shear $\sigma_{\mu\nu}$, and four-velocity u^μ, the expansion evolves as:

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu. \tag{5}$$

In Einstein's theory, the Ricci curvature is related to the stress-energy tensor by:

$$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right). \tag{6}$$

For attractive gravity ($G > 0$) and positive energy density (Assumption B), the term $R_{\mu\nu}u^\mu u^\nu$ is generically positive, leading to:

$$\frac{d\theta}{d\tau} < 0. \tag{7}$$

This describes *geodesic focusing*: nearby worldlines converge, forming caustics and enabling bound structures.

Reversing the sign of G inverts this behavior:

$$\frac{d\theta}{d\tau} > -\frac{1}{3}\theta^2 - \sigma^2. \tag{8}$$

The curvature term now opposes focusing. Repulsive gravity converts spacetime into a *defocusing geometry*, eliminating caustics, bound congruences, and stable worldline neighborhoods.

This is not a statement about specific solutions but about the global character of geodesic flow. A defocusing spacetime cannot sustain the bundles of worldlines necessary for persistent physical structures.

5 Physical Consequences

The geometric obstruction manifest in Eq. (5) has immediate physical consequences:

- **No Bound Orbits:** As shown in the Newtonian limit, closed trajectories require attractive potentials. Repulsive gravity eliminates all such solutions.
- **No Gravitational Lensing:** Light bending requires geodesic convergence. A defocusing geometry produces only divergence, incompatible with observed lensing phenomena.
- **No Stable Structures:** Extended objects—stars, planets, galaxies—rely on gravitational self-binding. Without attraction, no equilibrium configurations exist.
- **No Thermodynamic Gradients:** Entropy production and information processing require localized energy concentrations, which cannot form in a uniformly repulsive field.
- **No Long-Lived Observers:** Any physical system capable of observation requires structural persistence over time. Repulsive gravity precludes this.

A universe governed by repulsive gravity admits no persistent classical structures and therefore no physical observables in the usual sense. This is not merely a shift in phenomenology but a breakdown of the framework itself.

6 Discussion: Why This Is Not an Assumption

The attractive sign of gravity is typically treated as an empirical input: we observe that masses attract, and we encode this in the sign of the gravitational constant. Our analysis shows that this perspective is incomplete.

The attractive sign is constrained not by observation alone but by the requirement of *internal consistency* within variational spacetime theories. Given positive energy density and geodesic motion, the repulsive case fails to produce a coherent physical theory. It does not describe an alternative universe; it describes no universe at all.

This is analogous to the role of causality in field theory or unitarity in quantum mechanics. These are not arbitrary choices but structural necessities that emerge from the coherence of the framework. Similarly, attractive gravity emerges as a consistency condition rather than a model choice.

The attractive sign of gravity is therefore not a contingent feature selected by experiment alone, but a necessary condition for the internal consistency of variational spacetime dynamics with positive energy.

7 Conclusion

We have shown that reversing the sign of gravity does not yield an alternative physical theory but removes the mathematical possibility of bound structure altogether. Attractive gravity emerges as a consistency requirement rather than a model choice. This observation clarifies a commonly implicit assumption underlying all viable gravitational theories.

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